SCATTERING CALCULATIONS USING THE BOUNDARY ELEMENT METHOD

M.H. Lean, G. Jeng and A. Wexler
Department of Electrical Engineering
University of Manitoba
Winnipeg, Canada R3T 2N2

ABSTRACT

The use of isoparametric finite-elements, as a viable replacement for existing practices in the geometrical modelling of scattering surfaces, is presented. The boundary integral approach, together with the variational procedure, allows scattering parameters to be computed accurately.

Introduction

The isoparametric finite-element technique allows higher-order modelling with non-planar elements and results in reduced geometrical modelling error (Fig. 1). Previous alternatives have involved wire grids or patch representations [1,2] such that the scattering object and surface currents are unnecessarily poorly represented. This technique is applicable to the solution of vector integral equations pertaining to scattering problems [3].

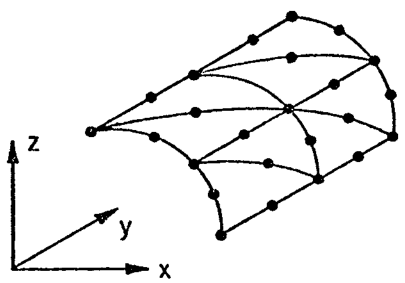

Figure 1: A boundary integral surface modelled by elements of second degree. Node points are indicated.

With the variational approach, theoretical convergence is guaranteed and it has been shown [4] that significantly fewer dependent variables are required to attain the same degree of accuracy as the point-matching method. Although the cost of generating the variationally-derived equations may be quite large, economies may be achieved through algorithmic improvements.

The fundamental problem in scattering computation is the explicit determination of the far-field parameters which depend on the configuration and material of the scatterer; the frequency, polarization and angle of incidence of the incidence field; and the far field observation angle. The sequence of computation follows from determination of the surface current densities at each node point on the surface of the scattering body. The scattered far field radiated by the currents are then ascertained and related to the radar cross section (RCS).

Theoretical Overview

Consider the case of a perfectly conducting obstacle subjected to an incident plane electromagnetic wave. Satisfying the boundary conditions imposed on the surface of the scatterer,

$$\hat{n} \times \bar{E} = 0 \qquad \hat{n} \cdot \bar{H} = 0 \tag{1}$$

which describes an exterior problem since no fields exist within the body. The free-space Green's function allows the formulation of the magnetic field integral equation (MFIE)

$$\bar{J}(s) = 2\hat{n} \times \bar{H}_{inc}(s) + \frac{1}{2\pi}\hat{n} \times \int_S \bar{J} \times \nabla'\phi ds' \tag{2}$$

which is a Fredholm integral of the second kind. Here, $\bar{J}(s)$ is the surface current density, $\bar{H}_{inc}(s)$ is the incident magnetic field and $\hat{n}$ is a unit normal on the surface and in an outward direction. Primed and unprimed coordinates refer to source and observation parameters respectively. The Rayleigh-Ritz discretization procedure allow $\bar{J}(s)$ to have finite-valued components at chosen points

$$\bar{J}(s) = \underline{\alpha}^T(s)\underline{\bar{J}} \tag{3}$$

where $\underline{\alpha}$ and $\underline{\bar{J}}$ are column matrices and the α_i are the polynomic interpolatory functions with

$$\alpha_i = \begin{cases} 1 \text{ at node } i \\ 0 \text{ at all other nodes} \end{cases} \tag{4}$$

In effect, $\bar{J}(s)$ is expressed as a linear combination of interpolatory polynomials. Substituting (3) into (2) and rearranging gives

$$\underline{\alpha}^T(s)\underline{J} - \frac{1}{2\pi}\hat{n} \times \int_S \underline{\alpha}^T(s')\underline{J} \times \nabla'\phi ds' = 2\hat{n} \times \bar{H}_{inc}(s) \tag{5}$$

Premultiplying by $\underline{\alpha}(s)$ and integrating over ds,

$$\int_S \underline{\alpha}(s)\underline{\alpha}^T(s) ds \underline{\bar{J}} - \frac{1}{2\pi}\hat{n} \times \int_S \underline{\alpha}(s) \int_S \underline{\alpha}^T(s')\underline{J} \times \nabla'\phi ds' ds$$

$$= \int_S 2\underline{\alpha}(s)\hat{n} \times \bar{H}_{inc}(s) ds \tag{6}$$

Performing the cross-products and expanding into component forms, (6) may be written in matrix form as

$$S\,\underline{\bar{J}} = \underline{b} \tag{7}$$

By using the Jacobian of transformation, a mapping from the local to the global coordinates is achieved such that the entries of the S and $\underline{b}$ matrices may be evaluated.

The scattered field is evaluated using the expression

$$\bar{H}^s(r) = \frac{e^{-jkr}}{4\pi r}\int_S (jk\bar{J} \times \hat{r}) e^{jk\hat{r}\cdot\bar{r}'} ds' \tag{8}$$

In particular, the x-component is given by

$$H_x^s = \frac{e^{-jkr}}{4\pi r}\int_S \left(\frac{jk}{r}(J_y z - J_z y)\right) e^{jk\frac{xx' + yy' + zz'}{r}} ds' \quad (9)$$

From (8) and (9), the RCS of a scatterer in a given orientation may be calculated from

$$\text{RCS} = \lim_{r\to\infty} 4\pi r^2 \frac{|\bar{H}^s|^2}{|\bar{H}_{inc}|^2} \quad (10)$$

The scattered field $\bar{H}^s$ and RCS are functions of angular coordinate.

Singularity of the Kernel

Double-surface integrations over the same element produces a singularity as the inner integral is singular at the Gaussian points of the outer integral. To cater for this, the singularity is extracted and integrated analytically. The singularity at $\bar{r} = \bar{r}'$ is removed by expressing the inner integral as

$$\int_S K(\bar{r}|\bar{r}')\alpha_j(\bar{r}')ds_{\bar{r}'}$$

$$= \int_S K(\bar{r}|\bar{r}')(\alpha_j(\bar{r}') - \alpha_j(\bar{r}))ds_{\bar{r}'}$$

$$+ \alpha_j(\bar{r})\int_S K(\bar{r}|\bar{r}')ds_{\bar{r}'} \quad (11)$$

The first integral on the right is evaluated numerically while the second has to be integrated analytically. Usually an expression for the latter does not exist for integration in closed form over curved surfaces. One alternative is to perform the integration over planar subregions. The approximation appears justifiable and the error introduced diminishes with the degree of approximation.

Conclusion

As a consequence of a number of numerical results, it appears that significant economies and/or improved accuracy are to be realized by application of the isoparametric boundary element method to the solution of electromagnetic scattering problems.

References

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